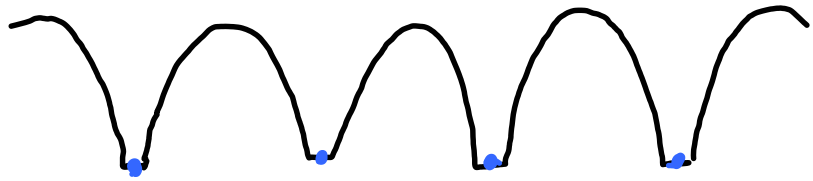
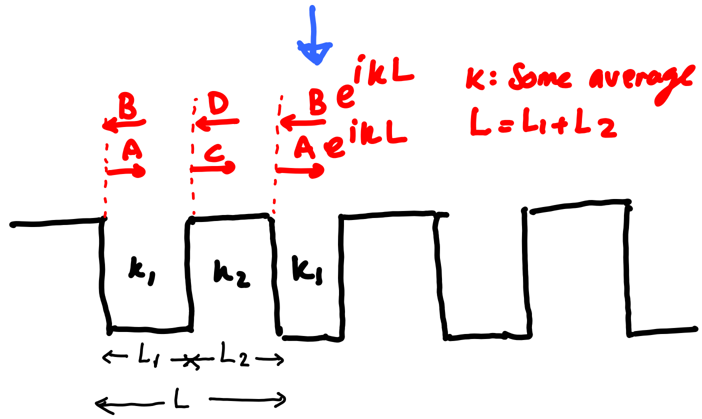
Kronig-Penney Model

In Solid:



Approximate with:



k: Some average k for the whole solid

$L = L_1 + L_2$

$$\begin{bmatrix} A \\ B \end{bmatrix} = P_{1free} P_{Step} \begin{bmatrix} C \\ D \end{bmatrix} = \frac{1}{2} \begin{bmatrix} (1+\frac{k_2}{k_1})e^{-ik_1L} & (1-\frac{k_2}{k_1})e^{-ik_1L} \\ (1-\frac{k_2}{k_1})e^{ik_1L} & (1+\frac{k_2}{k_1})e^{ik_1L} \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix}$$

$$\begin{bmatrix} C \\ D \end{bmatrix} = P_{2free} P_{2step} \begin{bmatrix} Ae^{ikL} \\ Be^{ikL} \end{bmatrix} = \frac{1}{2} \underbrace{\begin{bmatrix} (1+\frac{k_1}{k_2})e^{-ik_2L} & (1-\frac{k_1}{k_2})e^{-ik_2L} \\ (1-\frac{k_1}{k_2})e^{ik_2L} & (1+\frac{k_1}{k_2})e^{ik_2L} \end{bmatrix} \begin{bmatrix} e^{ikL} & 0 \\ 0 & e^{ikL} \end{bmatrix}}_{P'} \begin{bmatrix} A \\ B \end{bmatrix}$$

$$= \begin{bmatrix} P'_{11} & P'_{12} \\ P'_{21} & P'_{22} \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix}$$

$$\begin{cases} A - P_{11}C - P_{12}D = 0 \\ B - P_{21}C - P_{22}D = 0 \\ P'_{11}A + P'_{12}B - C = 0 \\ P'_{21}A + P'_{22}B - D = 0 \end{cases}$$

To have a nontrivial solution for A, B, C, D, the determinant of the coefficient must be zero:

$$\begin{vmatrix} 1 & 0 & -P_{11} & -P_{12} \\ 0 & 1 & -P_{21} & -P_{22} \\ P'_{11} & P'_{12} & -1 & 0 \\ P'_{21} & P'_{22} & 0 & -1 \end{vmatrix} = 0$$

$$= 1 - P'_{12}P_{21} - P'_{22}P_{22} - P'_{11}P_{11} + P'_{11}P'_{22}P_{11}P_{22} - P'_{11}P'_{22}P_{21}P_{12}$$

$$- P'_{21}P_{12} - P'_{21}P'_{12}P_{11}P_{22} + P'_{21}P'_{12}P_{21}P_{12} = 0$$

Replace P's and simplify to get:

$$\underbrace{\frac{k_2^2 - k_1^2}{2k_1k_2}\sin k_2L_2 \sin k_1L_1 + \cos k_2L_2 \cos k_1L_1}_{f(E)} = \cos kL$$

note: $E = \dfrac{\hbar^2 k_1^2}{2m}$

$E - V_0 = \dfrac{\hbar^2 k_2^2}{2m}$

we must have $f(E) < 1$ for the solution.